

DTP 99-75
EHU-FT/9912
hep-th/9911007

Composite diholes and intersecting brane–antibrane configurations in string/M-theory

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Abstract

We construct new configurations of oppositely charged, static black hole pairs (diholes) in four dimensions which are solutions of low energy string/M-theory. The black holes are extremal and have four different charges. We also consider diholes in other theories with an arbitrary number of abelian gauge fields and scalars, where the black holes can be regarded as composite objects. We uplift the four-charge solutions to higher dimensions in order to describe intersecting brane-antibrane systems in string and M-theory. The properties of the strings and membranes stretched inbetween these branes and antibranes are studied. Several other generic features of these solutions are discussed.

November 1999

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1 Introduction

The dual life of D-branes—on the one hand, as objects described by the conformal field theory of strings that end on them, and on the other hand as supergravity solitons—is one of the most interesting outcomes of the so-called “second string revolution.” Indeed, this duality lies at the heart of the AdS/CFT correspondence [1], and of the microscopic calculations of black hole entropy [2]. However, the progress in the construction of exact solutions for self-gravitating brane configurations lags far behind our understanding of those same configurations in string perturbation theory. To mention one outstanding example, we are still far from a satisfactory description of self-gravitating localized brane intersections, despite much effort and some progress in certain cases [3].

Supersymmetric D-branes of type II theories are well-studied examples of BPS states, which form a very special class of states in the Hilbert space of string and field theories with extended supersymmetry. So far, most non-perturbative tests of the duality conjecture are based on such BPS states, mainly because they are stable, protected from quantum radiative corrections, and hence, easier to handle. However nice their properties though, these states do not account for the full spectrum of any string theory, and the study of non-BPS states, which is still in its infancy, is essential in any viable attempt at shaping up M-theory.

In the same way as one may interpret BPS D-branes as classical solutions to type II supergravities, there have been some efforts to construct supergravity solutions describing a particular class of non BPS states corresponding to D-brane—anti-D-brane configurations. For instance, the four-dimensional Kaluza-Klein dipole constructed in [4] can be embedded in eleven-dimensional supergravity in order to provide a static $D6-\overline{D6}$ brane configuration of type IIA string theory suspended in an external magnetic field [5, 6]. More recently, exact solutions to Einstein-Maxwell theory, with and without dilaton, describing static (but unstable) pairs of extremal black holes with opposite charges (hereafter, *diholes*) were constructed in a background magnetic field, and were argued to admit an interpretation in terms of a system of intersecting branes and intersecting antibranes in higher dimensions, after a suitable uplifting of the four-dimensional solutions when the dilaton coupling takes one of four special values [7]. The explicit task of uplifting the solutions in this way has been undertaken since then in [8]. Other recent studies of configurations of this type in the context of string theory include [9].

The generalization of these configurations to the case where the charges of the branes are not equal is the main subject of this paper. After reviewing in Section 2 the known case of single charge diholes, we construct in Section 3 a new exact solution of four-dimensional General Relativity describing oppositely charged, static black hole pairs, where the black holes are extremal and have an arbitrary number n of different charges. Black holes of this sort can be regarded as composites of n extremal, singly charged black holes. Therefore, our solutions

describe composites of n diholes. We however mostly concentrate on the case $n = 4$ with two electric and two magnetic charges, because of the well-known consistent truncation of a wide class of low energy superstring theory compactifications to a four-dimensional action whose bosonic sector,

$$I = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left\{ R - \frac{1}{2} [(\partial\eta)^2 + (\partial\sigma)^2 + (\partial\rho)^2] - \frac{e^{-\eta}}{4} [e^{-\sigma-\rho} F_{(1)}^2 + e^{-\sigma+\rho} F_{(2)}^2 + e^{\sigma+\rho} F_{(3)}^2 + e^{\sigma-\rho} F_{(4)}^2] \right\}, \quad (1.1)$$

is the starting point of our analysis when considering four abelian gauge fields. Our construction is inspired by two known results. First of all, when the branes carry different charges, the corresponding four-dimensional black holes appear as solutions to theories with four $U(1)$ gauge fields and three scalars of type (1.1) [10]. Second of all, four-charge pairs of black holes accelerating apart were found in [11, 12], which, when adequately written, are very suggestive of the form their static counterparts might take. (Composites with two charges, which can be obtained as a particular case of the four-charge case, are also of interest, see [13, 14]).

As with other diholes, the composite dihole solutions in an asymptotically flat space suffer from conical singularities along the axis of symmetry. These singularities can be removed by suspending the diholes in external magnetic fields, a procedure we will examine in some detail. It should be noted that, for reasons similar to those discussed in [7], composite diholes are unstable equilibrium configurations.

Although these new composite diholes deserve to be studied in their own right as four-dimensional solutions of General Relativity, they can be embedded in ten or eleven-dimensional supergravities, and interpreted as systems of intersecting branes and intersecting anti-branes. This is discussed in Section 4, in an attempt to test any relevant connections between supergravity solitons and non BPS states described by brane—anti-brane type of configurations, and therefore to shed some light on the duality conjecture beyond the BPS analysis.

2 Single-charge diholes

We start by reviewing the single-charge dihole solutions as they were studied in [7]. Our purpose here is not only to introduce the structure of the solutions, which is somewhat involved, but also to discuss the solutions in a coordinatization that is slightly different from the one used in previous literature, and which will be required later.

It was shown in [15, 16] that the field equations of Einstein-Maxwell-Dilaton theory derived from the action

$$I = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left(R - 2(\partial\phi)^2 - e^{-2\alpha\phi} F^2 \right), \quad (2.1)$$

admit solutions with metric

$$ds^2 = \left(\frac{\Delta + a^2 \sin^2 \theta}{\Sigma} \right)^{\frac{2}{1+\alpha^2}} \left[-dt^2 + \frac{\Sigma^{\frac{4}{1+\alpha^2}}}{(r^2 - \gamma^2 \cos^2 \theta)^{\frac{3-\alpha^2}{1+\alpha^2}}} \left(\frac{dr^2}{\Delta} + d\theta^2 \right) \right] + \frac{\Delta \sin^2 \theta}{\left(\frac{\Delta + a^2 \sin^2 \theta}{\Sigma} \right)^{\frac{2}{1+\alpha^2}}} d\varphi^2, \quad (2.2)$$

dilaton

$$e^{-\phi} = \left(\frac{\Delta + a^2 \sin^2 \theta}{\Sigma} \right)^{\frac{\alpha}{1+\alpha^2}}, \quad (2.3)$$

and (magnetic) one-form gauge potential,

$$A = \frac{2}{\sqrt{1+\alpha^2}} \frac{M(r+M)a}{\Delta + a^2 \sin^2 \theta} \sin^2 \theta d\varphi, \quad (2.4)$$

where

$$\begin{aligned} \Delta &= r^2 - \gamma^2, \\ \Sigma &= (r+M)^2 - a^2 \cos^2 \theta. \end{aligned} \quad (2.5)$$

The parameter

$$\gamma^2 \equiv M^2 + a^2 \quad (2.6)$$

has been introduced here for later convenience. Actually, at fixed value of the dilaton coupling α , the two parameters defining the solution can be taken to be γ and a .

When comparing these solutions to those in [7, 15, 16] and to the literature on similar solutions, it is important to notice a relative shift of the radial coordinate r . To recover the solutions in the form given in those papers, let

$$r \rightarrow r - M. \quad (2.7)$$

A solution with electric field, dual to the magnetic solution above, can be readily constructed. In this case, the dilaton changes from ϕ to $-\phi$, and the electric gauge potential becomes

$$A = -\frac{2}{\sqrt{1+\alpha^2}} \frac{Ma \cos \theta}{\Sigma} dt. \quad (2.8)$$

We now follow the analysis in [7]. The solution is clearly asymptotically flat as $r \rightarrow \infty$, and in this asymptotic region the gauge field is that of a dipole. Although the solution contains apparent singularities at $r = \gamma$ (where $\Delta = 0$), the actual situation is somewhat subtle. Notice first that the axis of symmetry of the solution (the fixed-point set of the Killing vector ∂_φ) consists of the semi-infinite lines $\theta = 0, \pi$ (running from $r = \gamma$ to ∞), and the segment $r = \gamma$

that stretches in between them (running from $\theta = 0$ to π). The crucial feature of these solutions is that at each of the poles ($r = \gamma, \theta = 0$) and ($r = \gamma, \theta = \pi$), lies a (distorted) extremal charged dilatonic black hole. In order to see this, change coordinates r, θ to $\rho, \bar{\theta}$,

$$\begin{aligned} r &= \gamma + \frac{\rho}{2}(1 + \cos \bar{\theta}) , \\ \sin^2 \theta &= \frac{\rho}{\gamma}(1 - \cos \bar{\theta}) , \end{aligned} \quad (2.9)$$

and examine the solution for small values of ρ . On doing so, the metric in this region takes the form

$$ds^2 \rightarrow g^{\frac{2}{1+\alpha^2}}(\bar{\theta}) \left[- \left(\frac{\rho}{q} \right)^{\frac{2}{1+\alpha^2}} dt^2 + \left(\frac{q}{\rho} \right)^{\frac{2}{1+\alpha^2}} (d\rho^2 + \rho^2 d\bar{\theta}^2) \right] + \left(\frac{q}{\rho} \right)^{\frac{2}{1+\alpha^2}} \frac{\rho^2 \sin^2 \bar{\theta}}{g^{\frac{2}{1+\alpha^2}}(\bar{\theta})} d\varphi^2 , \quad (2.10)$$

with

$$g(\bar{\theta}) = \cos^2(\bar{\theta}/2) + \frac{a^2}{\gamma^2} \sin^2(\bar{\theta}/2) , \quad (2.11)$$

and

$$q \equiv \frac{M(\gamma + M)}{\gamma} . \quad (2.12)$$

For $\alpha = 0$ (the case of Einstein-Maxwell theory without a dilaton), this geometry is that of a Bertotti-Robinson universe ($AdS_2 \times S^2$), albeit distorted by the factor $g(\bar{\theta})$. This is, we find a geometry just like that of the region close to the horizon ($\rho = 0$) of an extremal Reissner-Nordström black hole, but, instead of being spherically symmetric, it is elongated along the axis in a prolate shape. For other values of α the solution at $\rho = 0$ has a curvature singularity which is just like the one at the core of extremal charged dilaton black holes, although, again, the geometry is not spherically symmetric due to the distorting factor $g(\bar{\theta})$. Hence we see that the dipolar field of the full solution is created by two oppositely charged extremal black holes: a *dihole*¹.

That the dipolar field is originated by a pair of extremal black holes, and not by, say, a pointlike or linear singularity or a pair of charges of a different kind, is obviously a non trivial issue. Apparently, Bonnor's dipole (*i.e.*, the $\alpha = 0$, non-dilatonic solution found in [15]) was originally thought to describe a singular pointlike (or segment-like) dipole. The first identification of a self-gravitating pole-antipole configuration was made for the case of Kaluza-Klein theory ($\alpha = \sqrt{3}$) in [4], and then refined in [5]. However, in those papers the interpretation was made on the basis of topological arguments that are particular to the higher-dimensional structure of Kaluza-Klein theory, and which cannot be applied to solutions with other values of

¹In [16] it was argued that for the case $\alpha = 1$, and only for this case, the solution contains regular non-extremal horizons, and describes a black hole-white hole configuration. This interpretation is not consistent with what we have just described: two extremal horizons, regular for $\alpha = 0$ and singular for $\alpha > 0$.

the dilaton. The Kaluza-Klein dipole was later analyzed in [6] by means of essentially the same transformation as (2.9). That the solutions (2.2) of [16] actually describe a dihole for arbitrary values of α , including the case $\alpha = 0$ which has regular horizons, was first proven in [7].

Given the two-black hole interpretation, it would be natural to expect the dihole solutions to contain the single black hole solutions as the limiting case where one of the holes is pulled infinitely away from the other. This is indeed the case. Working in coordinates $\rho, \bar{\theta}$, if the parameter a is taken to infinity while keeping all other quantities finite, then the solution reduces precisely to that of a single extremal dilatonic black hole. The parameter a plays then the role of a measure of the separation between the holes. However, this is just a qualitative statement, since the proper spatial distance between the extremal horizons (for $\alpha = 0$) is actually infinite. A more accurate statement is to say that increasing a , while keeping the holes' charge fixed, increases the value of the dipole moment (the relation, however, becomes approximately linear only for large a).

We are primarily interested, however, in the situation where both black holes are present in the solution, and therefore we consider finite values of a . The attraction, gravitational and electromagnetic, that they exert on one another is not balanced by any external field, so the geometry reacts, as is usual in these situations, by producing conical singularities along the symmetry axis [7, 16]. On physical grounds, it is clear that an external magnetic field aligned with the dihole could provide the force to balance the configuration. An exact solution containing such a field can be constructed by applying a Harrison transformation to (2.2). This was done in [7], and results in the metric

$$ds^2 = \Lambda^{\frac{2}{1+\alpha^2}} \left[-dt^2 + \frac{\Sigma^{\frac{4}{1+\alpha^2}}}{(r^2 - \gamma^2 \cos^2 \theta)^{\frac{3-\alpha^2}{1+\alpha^2}}} \left(\frac{dr^2}{\Delta} + d\theta^2 \right) \right] + \frac{\Delta \sin^2 \theta}{\Lambda^{\frac{2}{1+\alpha^2}}} d\varphi^2, \quad (2.13)$$

the dilaton, $e^{-\phi} = \Lambda^{\frac{\alpha}{1+\alpha^2}}$, and the gauge potential,

$$A = \frac{\frac{2}{\sqrt{1+\alpha^2}} Ma(r+M) + \frac{1}{2} B [((r+M)^2 - a^2)^2 + \Delta a^2 \sin^2 \theta]}{\Lambda \Sigma} \sin^2 \theta d\varphi, \quad (2.14)$$

with

$$\Lambda = \frac{1}{\Sigma} \left\{ \Delta + a^2 \sin^2 \theta + 2\sqrt{1+\alpha^2} BM(r+M)a \sin^2 \theta + \frac{1+\alpha^2}{4} B^2 \sin^2 \theta \left[((r+M)^2 - a^2)^2 + \Delta a^2 \sin^2 \theta \right] \right\}, \quad (2.15)$$

and Δ and Σ still given by (2.5). At distances much larger than the size of the dipole ($r \gg a, M$) the solution asymptotes to the dilatonic Melvin universe, which describes a self-gravitating, cylindrically symmetric magnetic field. It is of interest now to examine what effect the external field has on the black hole horizons. To see this, change coordinates again as in (2.9), and then

focus on small values of ρ . One finds a geometry just like (2.10), but now the deformation function is

$$g(\bar{\theta}) = \cos^2(\bar{\theta}/2) + \left(\frac{a}{\gamma} + Bq\sqrt{1+\alpha^2} \right)^2 \sin^2(\bar{\theta}/2) , \quad (2.16)$$

q being the parameter defined in (2.12). Notice that if we tune the external magnetic field to the value

$$B = \frac{1}{q\sqrt{1+\alpha^2}} \left(1 - \frac{a}{\gamma} \right) = \frac{1}{\sqrt{1+\alpha^2}} \frac{\gamma - a}{M(\gamma + M)} , \quad (2.17)$$

then $g(\bar{\theta}) = 1$, and the distortion of the holes disappears². It was shown in [7] that this very same value of the magnetic field produces a cancellation of the conical defects along the symmetry axis, with the choice $\Delta\varphi = 2\pi$. It is quite peculiar that the horizons recover their spherical symmetry precisely when the forces in the system are balanced and the two black holes are suspended in (unstable) equilibrium in the external field. In a sense, the latter exactly compensates for the distortions of the horizon induced by the presence of the other hole.

In the dual electric case the background Melvin field cannot be introduced by means of a solution-generating transformation as in the magnetic case. Nevertheless, the electric solution can be constructed by straightforward dualization of the magnetic one. The dilaton reverses sign as usual, and we find the electric potential to be

$$A = \left[B \cos \theta \left(r - 2M + \frac{aBM\sqrt{1+\alpha^2}}{2} (2 + \sin^2 \theta) \right) - \frac{2Ma \cos \theta (1 - \frac{1}{2}aB\sqrt{1+\alpha^2} \sin^2 \theta)^2}{\sqrt{1+\alpha^2}\Sigma} \right] dt. \quad (2.18)$$

This form of the potential manifestly shows how the potential tends to a “uniform” field $A_t \rightarrow Br \cos \theta$ as $r \rightarrow \infty$. In this case B is the asymptotic electric field along the axis.

The physical charge of the holes can be easily read by examining the gauge potentials in the region near the horizons. If we keep the field B arbitrary, instead of fixing it to the equilibrium value (2.17), then, as $\rho \rightarrow 0$, the dilaton goes to

$$e^{-\phi} \rightarrow \left(\frac{\rho}{q} g(\bar{\theta}) \right)^{\frac{\alpha}{1+\alpha^2}} \quad (2.19)$$

so when the balance condition (2.17) is achieved the angular dependence disappears. The potential, in its turn, becomes

$$A \rightarrow \frac{q}{\sqrt{1+\alpha^2}} \left(\frac{a}{\gamma} + Bq\sqrt{1+\alpha^2} \right) \frac{1 - \cos \bar{\theta}}{g(\bar{\theta})} d\varphi , \quad (2.20)$$

²There is a second value of B that yields $g(\bar{\theta}) = 1$, but here we have chosen the one for which $B \rightarrow 0$ as $a \rightarrow \infty$.

or, in the electric case,

$$A \rightarrow -\frac{1}{\sqrt{1+\alpha^2}} \left(\frac{a}{\gamma} + Bq\sqrt{1+\alpha^2} \right) \frac{\rho}{q} dt, \quad (2.21)$$

(here we have gauged away a constant) with, of course, a reversal in the sign of the dilaton. From here we infer that the charge is,

$$Q = \frac{1}{\sqrt{1+\alpha^2}} \frac{q}{\frac{a}{\gamma} + Bq\sqrt{1+\alpha^2}} \frac{\Delta\varphi}{2\pi} \quad (2.22)$$

in either the electric or magnetic solutions.

3 Multi-charged diholes

We address now the construction of new dihole solutions in theories with richer field content than the single-gauge field theories of (2.1).

3.1 String/M-theory diholes with four charges

In this subsection we consider a theory containing four abelian gauge fields and three scalars, with action (1.1), which appears as a consistent truncation of a large variety of compactifications of low energy string theory, such as toroidally compactified heterotic, IIA and IIB string theories, and also $D = 11$ supergravity [8, 17, 18]. Correspondingly, there is a large number of possible higher dimensional interpretations of the different gauge fields and their charge sources. A few of all these possible oxidations will be discussed in Sec. 4.

Black hole solutions to this theory were constructed in [10]. The black holes carry charges Q_i , $i = 1 \dots, 4$ under each of the gauge fields, the charges Q_1, Q_3 being of magnetic type, and Q_2, Q_4 electric (or viceversa, if we consider a dual configuration). When only s out of the four possible charges are equal and non-zero, and the rest are zero, then the theory, and its solutions, reduce to those of the Einstein-Maxwell-dilaton theory with coupling $\alpha = \sqrt{(4-s)/s}$. This is, solutions with 1, 2, 3 or 4 equal charges correspond to dilaton coupling $\alpha = \sqrt{3}, 1, 1/\sqrt{3}$ and 0, respectively. The extremal black hole solutions can be constructed following the “harmonic function rule” (see *e.g.*, [19]). Each gauge field enters in the solution through products of harmonic functions, in a manner that does essentially not depend on the other gauge fields. In [11, 12] it was shown that solutions with two such black holes accelerating apart could also be found for these theories (see [14] for the solutions in a $U(1)^2$ theory). We are interested here in configurations where the two black holes with opposite charges are static.

We have managed to construct exact solutions to the field equations for these theories with $U(1)^4$ dipole fields. Their metric is

$$ds^2 = (T_1 T_2 T_3 T_4)^{1/2} \left[-dt^2 + \frac{\Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4}{(r^2 - \gamma^2 \cos^2 \theta)^3} \left(\frac{dr^2}{\Delta} + d\theta^2 \right) \right] + \frac{\Delta \sin^2 \theta}{(T_1 T_2 T_3 T_4)^{1/2}} d\varphi^2 . \quad (3.1)$$

Here

$$\Delta = r^2 - \gamma^2 , \quad (3.2)$$

$$\Sigma_i = (r + M_i)^2 - a_i^2 \cos^2 \theta , \quad (3.3)$$

$$T_i = \frac{\Delta + a_i^2 \sin^2 \theta}{\Sigma_i} , \quad i = 1, \dots, 4 . \quad (3.4)$$

The magnetic gauge potentials $A_{(1)}, A_{(3)}$, are given by

$$A_{(i)} = \frac{2a_i M_i (r + M_i) \sin^2 \theta}{\Delta + a_i^2 \sin^2 \theta} d\varphi , \quad i = 1, 3 \quad (3.5)$$

whereas the electric potentials $A_{(2)}, A_{(4)}$, are

$$A_{(i)} = -\frac{2a_i M_i \cos \theta}{\Sigma_i} dt , \quad i = 2, 4 . \quad (3.6)$$

The scalar fields, in turn, take the form

$$e^{-2\eta} = \frac{T_1 T_3}{T_2 T_4}, \quad e^{-2\sigma} = \frac{T_1 T_4}{T_2 T_3}, \quad e^{-2\rho} = \frac{T_1 T_2}{T_3 T_4}. \quad (3.7)$$

The solutions are parameterized in terms of five independent parameters. Physically, the parameters can be regarded as fixing the four charges of the holes and the “separation” between the pair. In practice, we will choose the independent parameters to be γ (which we take to be positive) and all the a_i (satisfying $|a_i| \leq \gamma$). The other parameters M_i are not independent, but rather given by

$$M_i^2 = \gamma^2 - a_i^2 . \quad (3.8)$$

When $a_i^2 = \gamma^2$ for all i , then all the M_i ’s vanish and the metric is that of flat space. In general we can have some $M_i = 0$ for some values of i , and nonzero for others, and get a non-trivial solution.

There are several non-obvious aspects in going from the solutions of the single-gauge field theory to the solutions of the $U(1)^4$ theory. One of them is that the combination $a_i^2 + M_i^2$ should take the same value for all i , so with our choice of radial coordinate, the function Δ is the same for all values of i . Another point is related to the characteristic way in which

the metric functions in (3.1) factorize into contributions from each separate gauge field (a similar factorization had been observed also for Melvin fields and accelerating black holes in [12]). The way the factorization happens in these solutions relies crucially on our choice of the radial coordinate, explained in the previous section. To see this, realize that when more than one parameter M_i is involved, the radial shift (2.7) cannot be properly undone³. The factorization suggests that the $U(1)^4$ dihole can then be thought of as a composite of four diholes. For an isolated $U(1)^4$ black hole it is possible to separate, at zero cost in energy, each of the constituents from the other three, *i.e.*, the black hole can be regarded as a composite of four marginally bound components [18]. However, it is not clear whether we can separate, at zero energy cost, the single-charge component diholes of a composite dihole. It may well be that what was in isolation a state bound at threshold (the four-charge extremal black hole) becomes non-marginally bound in the presence of its anti-state.

It is also a straightforward matter to check that the dilatonic dihole solutions (2.2) for $\alpha = \sqrt{3}, 1, 1/\sqrt{3}$ and 0 are recovered by taking 1, 2, 3 or 4 non-zero and equal values of M_i .

The analysis of these solutions can be done in exactly the same manner as we have done for Bonnor's dipole and its dilatonic counterparts. Coordinate singularities occur when $r = \gamma$, and these turn out to be, away from the poles, conical singularities. Again, a straightforward analysis of the conical deficits along the various portions of the symmetry axis reveals that it is not possible to eliminate the deficit along the segment $r = \gamma$ with the natural choice of period $\Delta\varphi = 2\pi$ which cancels the deficit along the lines $\theta = 0, \pi$. However, these singularities can be resolved by introducing magnetic background fields in our axisymmetric solutions by means of the generalized Harrison transformation constructed in [12] for the $U(1)^4$ theory, and by subsequently tuning them to a value which eliminates the conical deficit. The latter point, we will see, becomes somewhat subtle when more than one gauge field is present.

After applying the generalized Harrison transformation, the metric of the $U(1)^4$ dipole solution becomes

$$ds^2 = (\Lambda_1\Lambda_2\Lambda_3\Lambda_4)^{1/2} \left[-dt^2 + \frac{\Sigma_1\Sigma_2\Sigma_3\Sigma_4}{(r^2 - \gamma^2 \cos^2 \theta)^3} \left(\frac{dr^2}{\Delta} + d\theta^2 \right) \right] + \frac{\Delta \sin^2 \theta}{(\Lambda_1\Lambda_2\Lambda_3\Lambda_4)^{1/2}} d\varphi^2, \quad (3.9)$$

the magnetic and electric potentials are given by,

$$A_{(i)} = \frac{2a_i M_i (r + M_i) \sin^2 \theta + \frac{1}{2} B_i \sin^2 \theta \left[\left((r + M_i)^2 - a_i^2 \right)^2 + a_i^2 \Delta \sin^2 \theta \right]}{\Lambda_i \Sigma_i} d\varphi, \quad i = 1, 3 \quad (3.10)$$

³Notice that this implies that the “modified harmonic function” rule conjectured in [8] on the basis of the solutions with a single gauge field is of little use in trying to get to the new solutions with different charges, since that rule was based on a choice of radial coordinate that is inappropriate for this purpose.

$$A_{(i)} = \left[B_i \cos \theta \left(r - 2M_i + \frac{a_i B_i M_i}{2} (2 + \sin^2 \theta) \right) - \frac{2M_i a_i \cos \theta (1 - \frac{1}{2} a_i B_i \sin^2 \theta)^2}{\Sigma_i} \right] dt, \quad i = 2, 4 \quad (3.11)$$

and the scalars by,

$$e^{-2\eta} = \frac{\Lambda_1 \Lambda_3}{\Lambda_2 \Lambda_4}, \quad e^{-2\sigma} = \frac{\Lambda_1 \Lambda_4}{\Lambda_2 \Lambda_3}, \quad e^{-2\rho} = \frac{\Lambda_1 \Lambda_2}{\Lambda_3 \Lambda_4}, \quad (3.12)$$

where

$$\Lambda_i = \frac{\Delta + a_i^2 \sin^2 \theta + 2B_i a_i M_i (r + M_i) \sin^2 \theta + \frac{1}{4} B_i^2 \sin^2 \theta \left[(r + M_i)^2 - a_i^2 \right]^2 + a_i^2 \Delta \sin^2 \theta}{\Sigma_i}. \quad (3.13)$$

This solution obviously reduces to the previous one if we set $B_i = 0$. We have denoted the external fields collectively as B_i , even if for $i = 2, 4$ they are electric fields. Observe that the metric and scalars can be obtained from (3.1) and (3.7) by simply substituting Λ_i for T_i .

Along the outer semi-axes $\theta = 0, \pi$ the conical deficit is given by $\delta_{(\theta=0,\pi)} = 2\pi - \Delta\varphi$, no matter what the value of the external fields B_i is. We thus choose $\Delta\varphi = 2\pi$ in order to remove the conical deficit on that portion of the symmetry axis. On the other hand, the deficit along the inner segment of the axis, $r = \gamma$, is calculated to be,

$$\delta_{(r=\gamma)} = 2\pi - \prod_{i=1}^4 \left(\frac{a_i}{\gamma} + B_i q_i \right)^{-1} \Delta\varphi, \quad (3.14)$$

where we have defined

$$q_i \equiv \frac{M_i(\gamma + M_i)}{\gamma}. \quad (3.15)$$

We can see that, for the choice of period in the variable φ we made earlier, *i.e.*, $\Delta\varphi = 2\pi$, the conical singularity along $r = \gamma$ disappears when

$$\prod_{i=1}^4 \left(\frac{a_i}{\gamma} + B_i q_i \right) = 1. \quad (3.16)$$

A *very particular* solution of this equation is obtained by requiring each of the factors on the l.h.s. to be equal to 1, *i.e.*, set the strength of the external fields to the values

$$B_i = \frac{\gamma - a_i}{M_i(\gamma + M_i)} = \frac{2M_i}{(\gamma + M_i + a_i)^2}, \quad i = 1 \dots, 4 \quad (3.17)$$

which can be interpreted as a separate force-balance condition for each of the gauge fields. Nevertheless, it should be kept in mind that (3.17) is by no means a typical solution. On the contrary, the balance of forces will typically be achieved with different contributions from each

factor in (3.16). As a matter of fact, it is even possible to satisfy (3.16) for diholes with four different charges by turning on only one external field, say B_1 .

Although the metric still appears to be singular at the endpoints of the $U(1)^4$ dipole, one can actually reveal its true structure by studying the solution near the throat as we did in the previous section. As before, we may explore this region by changing coordinates from (r, θ) to $(\rho, \bar{\theta})$ as in (2.9) and by keeping ρ much smaller than any other scale in the problem. Near $r = \gamma, \theta = 0$, the metric becomes⁴,

$$ds^2 = g^2(\bar{\theta}) \left[-\frac{\rho^2}{q^2} dt^2 + \frac{q^2}{\rho^2} d\rho^2 + q^2 d\bar{\theta}^2 \right] + \frac{q^2 \sin^2 \bar{\theta}}{g^2(\bar{\theta})} d\varphi^2, \quad (3.18)$$

where $q = (q_1 q_2 q_3 q_4)^{1/4}$, and where $g(\bar{\theta}) = [g_1(\bar{\theta}) g_2(\bar{\theta}) g_3(\bar{\theta}) g_4(\bar{\theta})]^{1/4}$ with

$$g_i(\bar{\theta}) = \cos^2(\bar{\theta}/2) + \left(\frac{a_i}{\gamma} + B_i q_i \right)^2 \sin^2(\bar{\theta}/2) \quad (3.19)$$

a function such that $g_i(\bar{\theta}) = 1$ when the field B_i is tuned to the value (3.17). However, for more generic cases the deformation function $g(\bar{\theta})$ will be different from 1 even if the conical singularities are cancelled through (3.16), and the horizons will in general be deformed. This feature is particular to theories with more than one gauge field. In any event, we see that near the poles the solution exhibits, apart from the distortion, the same structure as a four-charge black hole near its horizon.

The gauge fields are also distorted near the throat, where they are given by

$$\begin{aligned} A_{(i)} &= \frac{q_i}{g_i(\bar{\theta})} \left(\frac{a_i}{\gamma} + B_i q_i \right) (1 - \cos \bar{\theta}) d\varphi, \quad i = 1, 3 \\ A_{(i)} &= -\frac{\rho}{q_i} \left(\frac{a_i}{\gamma} + B_i q_i \right) dt, \quad i = 2, 4 \end{aligned} \quad (3.20)$$

and the corresponding physical charges are

$$Q_i = \frac{\Delta\varphi}{2\pi} \frac{q_i}{\frac{a_i}{\gamma} + B_i q_i}. \quad (3.21)$$

Finally, the scalar fields in this limiting region are

$$e^{-2\eta} = \frac{q_2 q_4}{q_1 q_3} \frac{g_1(\bar{\theta}) g_3(\bar{\theta})}{g_2(\bar{\theta}) g_4(\bar{\theta})}, \quad e^{-2\sigma} = \frac{q_2 q_3}{q_1 q_4} \frac{g_1(\bar{\theta}) g_4(\bar{\theta})}{g_2(\bar{\theta}) g_3(\bar{\theta})}, \quad e^{-2\rho} = \frac{q_3 q_4}{q_1 q_2} \frac{g_1(\bar{\theta}) g_2(\bar{\theta})}{g_3(\bar{\theta}) g_4(\bar{\theta})}, \quad (3.22)$$

which present the unusual feature that, in general, they will vary as we move along the horizon.

⁴We are assuming here that all four charges are non-zero. The modifications for the case where some of them vanish can be inferred easily.

The deformation of the black hole horizons allows us to check a non-trivial aspect of the entropy-area law for black holes. Notice that when all four charges are turned on, the black holes have a non-singular, deformed horizon with non-vanishing area. Now, for an *isolated* extremal black hole the area is entirely determined by its physical charges Q_i as $\mathcal{A}_h = 4\pi\sqrt{Q_1Q_2Q_3Q_4}$. This area can be associated, through the Bekenstein-Hawking law, with an entropy. On physical grounds we would expect the entropy of the system to remain unchanged if its physical charges, which fix the state, remain fixed, no matter what the distortion of the horizon may be. It is by no means clear that the solutions given above should satisfy this property. Nonetheless, the area of each of the horizons in the dihole configuration is

$$\mathcal{A}_h = 4\pi\sqrt{q_1q_2q_3q_4} = 4\pi\sqrt{Q_1Q_2Q_3Q_4} , \quad (3.23)$$

where the last equality is obtained when the singularities are cancelled by requiring (3.16) (but not necessarily (3.17)) and $\Delta\varphi = 2\pi$. Hence, the area as a function of the physical charges remains unaltered, despite the deformation of the black hole horizon. A similar test of the invariance of the entropy under deformations of the horizon was performed in [12].

Finally, it is a straightforward exercise to show that when one of the holes is pulled away by making γ large, while keeping $r - \gamma$, $\gamma \sin^2 \theta$ and M_i finite, we get $\Sigma_i \simeq 2\gamma(\rho + M_i)$, $q_i \simeq M_i \simeq Q_i$ and $\Lambda_i \simeq (1 + \frac{Q_i}{\rho})^{-1}$, so that the metric becomes that of an isolated extremal $U(1)^4$ black hole.

3.2 $U(1)^n$ composite diholes

The results we have just described can be generalized to the following theories containing n gauge fields and $n - 1$ independent scalars with action

$$I = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left\{ R - \frac{1}{2n^2} \sum_{i=1}^n \sum_{j=i+1}^n (\partial\sigma_i - \partial\sigma_j)^2 - \frac{1}{n} \sum_{i=1}^n e^{-\sigma_i} F_{(i)}^2 \right\}, \quad (3.24)$$

and with the scalars satisfying

$$\sum_{i=1}^n \sigma_i = 0. \quad (3.25)$$

Such theories were considered in [12] as a generalization of the theories with four abelian gauge fields we have just considered. In general, these $U(1)^n$ theories do not seem to be related to low energy string/M-theory, nor to any other supergravity theory in four dimensions. Nevertheless they exhibit the same peculiarities as the $U(1)^4$ theories, which are merely a particular case of the above type of theory, as described in [12]. All these theories admit black hole solutions which follow the “harmonic function rule”, as well as solutions with two black holes accelerating apart [12]. It is therefore natural to expect that dihole solutions can be constructed as well.

Indeed, their metric is

$$ds^2 = \prod_{i=1}^n T_i^{2/n} \left[-dt^2 + \frac{\prod_{i=1}^n \Sigma_i^{4/n}}{(r^2 - \gamma^2 \cos^2 \theta)^3} \left(\frac{dr^2}{\Delta} + d\theta^2 \right) \right] + \frac{\Delta \sin^2 \theta}{\prod_{i=1}^n T_i^{2/n}} d\varphi^2, \quad (3.26)$$

where Δ , Σ_i and T_i take the same form as in (3.2),(3.3),(3.4). The potentials (in magnetic form) are

$$A_{(i)} = \frac{2a_i M_i (r + M_i) \sin^2 \theta}{\Delta + a_i^2 \sin^2 \theta} d\varphi, \quad i = 1, \dots, n \quad (3.27)$$

and the scalars

$$e^{-\sigma_i} = T_i^2 \prod_{j=1}^n T_j^{-2/n}. \quad (3.28)$$

The qualitative features of these solutions are precisely the same as for the $U(1)^4$ case of the previous subsection, so our discussion will be rather cursory. These solutions have $n+1$ independent parameters, $\{\gamma, a_i, i = 1, \dots, n\}$, while M_i are fixed by $M_i^2 = \gamma^2 - a_i^2$. By setting $n = 1$ and shifting the coordinate r to $r - M_1$, one recovers Bonnor's magnetic dipole solution of non-dilatonic Einstein-Maxwell theory [15]. The dilatonic solutions of [16] can also be recovered in a simple manner for rational values of α^2 [12]. To this effect, take s out of the n possible parameters M_i (say, $i = 1, \dots, s$) to be equal and non-zero, and the remaining M_i , $i = s+1, \dots, n$ to be vanishing. In this way, the solutions (2.2) are recovered, with dilaton coupling $\alpha = \sqrt{n/s - 1}$, and with the fields identified as

$$\begin{aligned} \sigma_1 &= \dots = \sigma_s = 2\alpha\phi, & \sigma_{s+1} &= \dots = \sigma_n = -\frac{2}{\alpha}\phi, \\ F_{(i)} &= \sqrt{\alpha^2 + 1} F, & i &= 1, \dots, s. \end{aligned} \quad (3.29)$$

The conical singularities that the solutions possess can be removed by means of the generalized Harrison transformation for the $U(1)^n$ theory constructed in [12], and by subsequently tuning them to a value which eliminates the conical deficit.

After applying the generalized Harrison transformation, the $U(1)^n$ dipole solution becomes,

$$ds^2 = \prod_{i=1}^n \Lambda_i^{2/n} \left[-dt^2 + \frac{\prod_{i=1}^n \Sigma_i^{4/n}}{(r^2 - \gamma^2 \cos^2 \theta)^3} \left(\frac{dr^2}{\Delta} + d\theta^2 \right) \right] + \frac{\Delta \sin^2 \theta}{\prod_{i=1}^n \Lambda_i^{2/n}} d\varphi^2, \quad (3.30)$$

$$e^{-\sigma_i} = \Lambda_i^2 \prod_{j=1}^n \Lambda_j^{-2/n}, \quad (3.31)$$

where the magnetic gauge potentials and Λ_i are given in (3.10) and (3.13).

All the features of the $U(1)^4$ solutions we described in the previous subsection carry over to the generic $U(1)^n$ case modulo some obvious adjustments.

4 Intersecting brane-antibrane configurations

We already pointed out that the solutions to the $U(1)^4$ theory admit embeddings into higher dimensional supergravity theories arising from string/M-theory at low energies. When uplifted to $D \geq 5$ dimensions, each individual charge (3.21) will typically be interpreted as the charge (density) of a p -brane, with all its spatial directions wrapped around a p -torus, and delocalized in $D - p - 4$ transverse directions. We are referring to branes in a manner loose enough to allow for pp-waves and KK monopoles to be introduced in a straightforward way in the discussion ⁵.

Solutions with just one out of four non-vanishing charges correspond to single brane solutions, whereas solutions with more than one charge describe brane intersections or (marginal) bound states of branes. A $U(1)^4$ dihole solution, when viewed in this way, can be oxidized to describe an intersection of up to four branes and an ‘anti-intersection’ of the corresponding anti-branes. We have tried to sketch such a configuration in Fig. 1. Each brane is parallel to its anti-brane, and the whole system is delocalized in such a way that the branes are localized in the overall transverse directions only.

The construction of brane-antibrane solutions based on the $U(1)^4$ dihole solutions described in Sec. 3 does not significantly differ in its concept from the way a one-black hole solution with four or less charges is uplifted to a brane configuration. Indeed, it was pointed out in [7] (see also [8]) that the diholes of Einstein-Maxwell-dilaton theory could be uplifted in a straightforward way to intersecting brane-anti-brane configurations of the sort just described, with the severe restriction that the charges of the intersecting branes should all be equal. This restriction can be relaxed when using our new $U(1)^4$ solutions, which allow for a richer catalog of configurations. The factorized form of the solutions, particularly that of the scalar fields involved, greatly helps in deducing the form of the higher dimensional (internal) metric components from those of ordinary brane configurations: the harmonic functions H_i of the latter get replaced by the functions T_i^{-1} or Λ_i^{-1} , whose inverses were introduced in (3.4) and (3.13). We stress that this rule, however, applies only to the internal dimensions and not to the four-dimensional part of the solution.

Note that we are unable to consider non-extremal branes, since a solution describing a pair of non-extremal charged black holes is not available. Let us also emphasize that solutions *without* an external field contain conical singularities along the symmetry axis, and a physical interpretation in string/M-theory in terms of, *e.g.*, local cosmic strings, is not clear. Nevertheless, it is possible to remove these singularities by introducing an external field, using a similar procedure to the one described in the previous sections.

⁵More precisely, any charge that would naturally enter a solution through a harmonic function can be paired up with its anticharge to provide a characterization of brane-antibrane systems

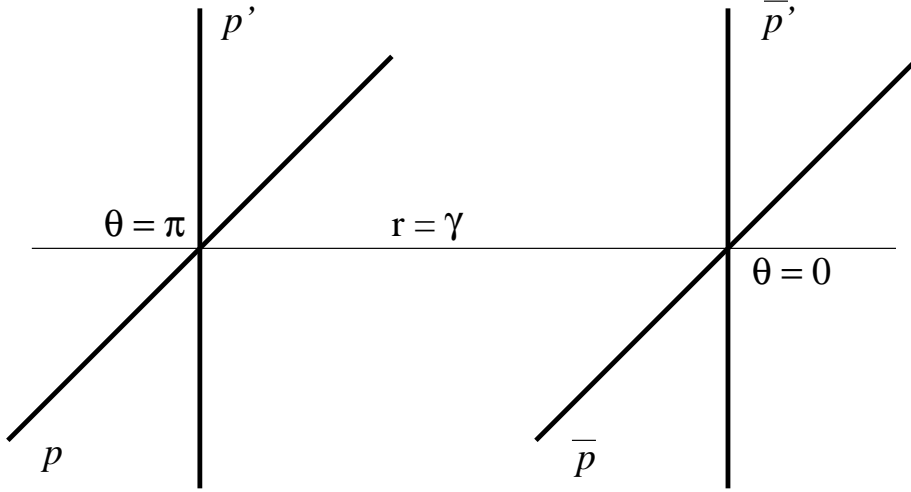


Figure 1: Geometry of the $p - p'$ brane and $\bar{p} - \bar{p}'$ brane intersections. The direction labelled p denotes directions along the p -brane which are transverse to the p' -brane, and viceversa. The same applies, mutatis mutandis, for the ‘anti-intersection’ $\bar{p} - \bar{p}'$. The p - and \bar{p} -branes are parallel. In the solutions described in the text the branes are delocalized in their relative transverse directions, and also in all but three overall transverse dimensions (r, θ, φ) . Of the latter directions, only the symmetry axis is shown in the figure. When infinitely separated from the other intersection ($\gamma \rightarrow \infty$), the p - and p' -branes we consider are marginally bound to each other.

4.1 Some explicit examples

We now illustrate the uplifting of our $U(1)^4$ dihole solutions to three different brane-antibrane configurations.

- (1) The $3 \perp 3 \perp 3 \perp 3$ and $\bar{3} \perp \bar{3} \perp \bar{3} \perp \bar{3}$ system in $D = 10$ type IIB theory.

Let us consider the ten-dimensional metric,

$$\begin{aligned}
 ds_{10}^2 = (T_1 T_2 T_3 T_4)^{1/2} & \left[-dt^2 + \frac{\Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4}{(r^2 - \gamma^2 \cos^2 \theta)^3} \left(\frac{dr^2}{\Delta} + d\theta^2 \right) \right] + \frac{\Delta \sin^2 \theta}{(T_1 T_2 T_3 T_4)^{1/2}} d\varphi^2 \\
 & + \left(\frac{T_1 T_2}{T_3 T_4} \right)^{1/2} dx_1^2 + \left(\frac{T_1 T_3}{T_2 T_4} \right)^{1/2} dx_2^2 + \left(\frac{T_1 T_4}{T_2 T_3} \right)^{1/2} dx_3^2 \\
 & + \left(\frac{T_2 T_3}{T_1 T_4} \right)^{1/2} dx_4^2 + \left(\frac{T_2 T_4}{T_1 T_3} \right)^{1/2} dx_5^2 + \left(\frac{T_3 T_4}{T_1 T_2} \right)^{1/2} dx_6^2. \quad (4.1)
 \end{aligned}$$

with the functions Σ_i and T_i defined in (3.3), (3.4). The ten-dimensional dilaton is constant, and the five-form field strength is given by,

$$F_{[5]} = dA_1 \wedge dx_4 \wedge dx_5 \wedge dx_6 + dA_2 \wedge dx_1 \wedge dx_4 \wedge dx_5$$

$$+dA_3 \wedge dx_1 \wedge dx_3 \wedge dx_5 + dA_4 \wedge dx_3 \wedge dx_5 \wedge dx_6. \quad (4.2)$$

with the magnetic potentials $A_{1/3}$ given in (3.5) and the electric potentials $A_{2/4}$ given in (3.6). We also use $r^2 = x_7^2 + x_8^2 + x_9^2$. The compactification of the above type IIB solution on a six-dimensional torus yields the $U(1)^4$ dipole solution (3.1). To check that the system indeed contains brane-antibrane pairs, one may change coordinates from (r, θ) to $(\rho, \bar{\theta})$ as in (2.9). In the limit where the parameter a_i is large, and where $\theta \rightarrow 0$, the function T_i becomes the inverse of the harmonic function of a delocalized D3-brane $T_i \rightarrow \tilde{T}_i = \left(1 + \frac{Q_i}{\rho}\right)^{-1}$, and the metric becomes

$$\begin{aligned} ds_{10}^2 = & \left(\tilde{T}_1 \tilde{T}_2 \tilde{T}_3 \tilde{T}_4\right)^{1/2} (-dt^2) + \left(\tilde{T}_1 \tilde{T}_2 \tilde{T}_3 \tilde{T}_4\right)^{-1/2} \left(d\rho^2 + \rho^2 (d\bar{\theta}^2 + \sin^2 \bar{\theta} d\varphi^2)\right) \\ & + \left(\frac{\tilde{T}_1 \tilde{T}_2}{\tilde{T}_3 \tilde{T}_4}\right)^{1/2} dx_1^2 + \left(\frac{\tilde{T}_1 \tilde{T}_3}{\tilde{T}_2 \tilde{T}_4}\right)^{1/2} dx_2^2 + \left(\frac{\tilde{T}_1 \tilde{T}_4}{\tilde{T}_2 \tilde{T}_3}\right)^{1/2} dx_3^2 \\ & + \left(\frac{\tilde{T}_2 \tilde{T}_3}{\tilde{T}_1 \tilde{T}_4}\right)^{1/2} dx_4^2 + \left(\frac{\tilde{T}_2 \tilde{T}_4}{\tilde{T}_1 \tilde{T}_3}\right)^{1/2} dx_5^2 + \left(\frac{\tilde{T}_3 \tilde{T}_4}{\tilde{T}_1 \tilde{T}_2}\right)^{1/2} dx_6^2. \end{aligned} \quad (4.3)$$

This is exactly the solution for the four D3-brane intersection constructed in [20, 21]. The solution for four anti-D3-branes intersection (i.e. for four D3-branes with opposite charge) is obtained when taking the $\theta \rightarrow \pi$ limit instead of the $\theta \rightarrow 0$ limit. However, in order to show that the system consists of the intersection of four D3-branes together with the intersection of four $\overline{D3}$ -branes in ten dimensions, one must consider finite values of a_i and take the near horizon limit of (4.1) in complete analogy with the four-dimensional case. As already mentioned earlier, this solution has a conical deficit along the symmetry axis, which pulls the branes apart from each other. However, as can be anticipated from the discussion of $U(1)^n$ diholes suspended in external magnetic fields, such a brane/anti-brane configuration can be cured of any conical deficit along the symmetry axis by tuning the magnetic field B to an appropriate value. The relevant ten-dimensional metric is then given by (4.1), where the four functions T_i are replaced by the functions Λ_i given in (3.13). The five-form field-strength is again formally written as in (4.2), with the magnetic potentials $A_{1/3}$ and electric potentials $A_{2/4}$ given by (3.10) and (3.11) respectively. In the near horizon limit, and with a_i large ($r \gg M_i$), the functions Λ_i , for $\theta \rightarrow 0$, become the harmonic functions \tilde{T}_i involved in the description of the four D3-brane intersections, and the limiting metric is (4.3). If $\theta \rightarrow \pi$ instead, one obtains the four $\overline{D3}$ -brane intersections. One concludes that the metric

$$\begin{aligned} ds_{10}^2 = & (\Lambda_1 \Lambda_2 \Lambda_3 \Lambda_4)^{1/2} \left[-dt^2 + \frac{\Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4}{(r^2 - \gamma^2 \cos^2 \theta)^3} \left(\frac{dr^2}{\Delta} + d\theta^2 \right) \right] + \frac{\Delta \sin^2 \theta}{(\Lambda_1 \Lambda_2 \Lambda_3 \Lambda_4)^{1/2}} d\varphi^2 \\ & + \left(\frac{\Lambda_1 \Lambda_2}{\Lambda_3 \Lambda_4} \right)^{1/2} dx_1^2 + \left(\frac{\Lambda_1 \Lambda_3}{\Lambda_2 \Lambda_4} \right)^{1/2} dx_2^2 + \left(\frac{\Lambda_1 \Lambda_4}{\Lambda_2 \Lambda_3} \right)^{1/2} dx_3^2 \\ & + \left(\frac{\Lambda_2 \Lambda_3}{\Lambda_1 \Lambda_4} \right)^{1/2} dx_4^2 + \left(\frac{\Lambda_2 \Lambda_4}{\Lambda_1 \Lambda_3} \right)^{1/2} dx_5^2 + \left(\frac{\Lambda_3 \Lambda_4}{\Lambda_1 \Lambda_2} \right)^{1/2} dx_6^2. \end{aligned} \quad (4.4)$$

is that of a system made of the intersection of four D3-branes and of the intersection of four $\overline{D3}$ -branes.

(2) The $2 \perp 2 \perp 5 \perp 5$ and $\bar{2} \perp \bar{2} \perp \bar{5} \perp \bar{5}$ system in $D = 11$ supergravity.

This system may be described by the $D = 11$ metric,

$$\begin{aligned}
ds_{11}^2 = & (T_2 T_4)^{2/3} (T_1 T_3)^{1/3} \left[-dt^2 + \frac{\Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4}{(r^2 - \gamma^2 \cos^2 \theta)^3} \left(\frac{dr^2}{\Delta} + d\theta^2 \right) \right] \\
& + \frac{\Delta \sin^2 \theta}{(T_2 T_4)^{1/3} (T_1 T_3)^{2/3}} d\varphi^2 + \left(\frac{T_2^2 T_1}{T_4 T_3^2} \right)^{1/3} dx_1^2 + \left(\frac{T_2^2 T_3}{T_4 T_1^2} \right)^{1/3} dx_2^2 \\
& + \left(\frac{T_4^2 T_1}{T_2 T_3^2} \right)^{1/3} dx_3^2 + \left(\frac{T_4^2 T_3}{T_2 T_1^2} \right)^{1/3} dx_4^2 + \left(\frac{T_1 T_3}{T_2 T_4} \right)^{1/3} (dx_5^2 + dx_6^2 + dx_7^2), \tag{4.5}
\end{aligned}$$

with four-form field strength,

$$\begin{aligned}
F_{[4]} = & 3dA_1 \wedge dx_2 \wedge dx_4 - 3dA_2 \wedge dx_1 \wedge dx_2 \\
& + 3dA_3 \wedge dx_1 \wedge dx_3 - 3dA_4 \wedge dx_3 \wedge dx_4. \tag{4.6}
\end{aligned}$$

The magnetic potentials $A_{1/3}$ are again defined by (3.5) and the electric potentials $A_{2/4}$ by (3.6). Here, $r^2 = x_8^2 + x_9^2 + x_{10}^2$. In the $(\rho, \bar{\theta})$ coordinate system, and in the limit of a_i large and $\theta \rightarrow 0$, $T_{2/4}^{-1}$ and $T_{1/3}^{-1}$ become the harmonic functions required for the description of electric M2 branes and magnetic M5 branes. The solution describes the system of intersecting branes (two M2 and two M5) and intersecting anti-branes (two $\overline{M2}$ branes and two $\overline{M5}$ -branes), and becomes the dipole solution in $U(1)^4$ theory when compactified on a seven-torus. Once again, we may construct an analog solution in the presence of an external magnetic field in order to remove any conical singularity arising in the above solution. Replacing T_i by Λ_i everywhere in (4.5), and taking the potentials to be (3.10) and (3.11), one obtains a solution to $D = 11$ supergravity which again describes a system of intersecting branes (two M2 and two M5) and intersecting anti-branes (two $\overline{M2}$ branes and two $\overline{M5}$ -branes), but this time with non-zero magnetic field B .

(3) The $2 \perp 2 \perp 2 \perp 6$ and $\bar{2} \perp \bar{2} \perp \bar{2} \perp \bar{6}$ system in $D = 10$ type IIA theory.

We end this subsection with a configuration we can relate to the $D6-\overline{D6}$ system studied by Sen, and which we will employ later in order to characterize the string stretching between the branes and anti-branes using arguments similar to those in [6].

The configuration is described by a solution to type IIA supergravity with metric

$$ds_{10}^2 = T_1^{1/8} (T_2 T_3 T_4)^{5/8} \left[-dt^2 + \frac{\Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4}{(r^2 - \gamma^2 \cos^2 \theta)^3} \left(\frac{dr^2}{\Delta} + d\theta^2 \right) \right] + \frac{\Delta \sin^2 \theta}{T_1^{7/8} (T_2 T_3 T_4)^{3/8}} d\varphi^2$$

$$+ \frac{T_1^{1/8} T_2^{5/8}}{T_3^{3/8} T_4^{3/8}} (dx_1^2 + dx_2^2) + \frac{T_1^{1/8} T_3^{5/8}}{T_2^{3/8} T_4^{3/8}} (dx_3^2 + dx_4^2) + \frac{T_1^{1/8} T_4^{5/8}}{T_2^{3/8} T_3^{3/8}} (dx_5^2 + dx_6^2) \quad (4.7)$$

and dilaton

$$e^{4\phi} = \frac{T_1^3}{T_2 T_3 T_4}, \quad (4.8)$$

with the functions Σ_i and T_i defined in (3.3), (3.4). Here, T_1 is associated with the $D6$ brane while $T_i, i = 2, 3, 4$ are associated with the three $D2$ branes. We first discuss in which context this metric corresponds to a $D6-\overline{D6}$ system. Setting to zero the charges q_2, q_3, q_4 of the three $D2$ branes, one indeed obtains the following $D6-\overline{D6}$ configuration,

$$ds_{10}^2 = T_1^{1/8} \left(-dt^2 + \sum_{i=1}^6 dx_i^2 \right) + T_1^{1/8} \Sigma_1 \left(\frac{dr^2}{\Delta} + d\theta^2 \right) + \frac{\Delta \sin^2 \theta}{T_1^{7/8}} d\varphi^2, \quad (4.9)$$

which coincides with the metric constructed following [5, 6] once the radial variable is shifted from $r \rightarrow r - M$ and the string frame is used (see *e.g.*, [8] for the explicit expression). Note that compactification on a T^6 torus yields the metric,

$$ds_4^2 = T_1^{1/2} \left[-dt^2 + \Sigma_1 \left(\frac{dr^2}{\Delta} + d\theta^2 \right) \right] + \frac{\Delta \sin^2 \theta}{T_1^{1/2}} d\varphi^2, \quad (4.10)$$

describing the Einstein-Maxwell dilatonic single charge dihole (2.2) when the coupling to the dilaton is $\alpha = \sqrt{3}$. Also, (4.9) may be uplifted to eleven dimensions to obtain

$$ds_{11}^2 = -dt^2 + \sum_{i=1}^6 dx_i^2 + \Sigma_1 \left(\frac{dr^2}{\Delta} + d\theta^2 \right) + T_1 (dx_{11} - 2A_\varphi d\varphi)^2 + \frac{\Delta \sin^2 \theta}{T_1} d\varphi^2, \quad (4.11)$$

where the magnetic potential is given by,

$$A_\varphi = \frac{a_1 M_1 (r + M_1) \sin^2 \theta}{\Delta + a_1^2 \sin^2 \theta}. \quad (4.12)$$

After shifting $r \rightarrow r - M$ in the above metric, one exactly recovers the Gross-Perry Kaluza-Klein dipole [4] embedded in eleven dimensions, which is the starting point of Sen's analysis of the $D6-\overline{D6}$ system.

We would like to stress at this point that, instead of performing a straightforward Kaluza-Klein compactification along the x_{11} direction in (4.11) to recover the configuration (4.9) (which possesses conical singularities along the axis), one may reduce along a twisted direction [5, 6]. In this way one obtains a configuration of $D6-\overline{D6}$ branes suspended in a magnetic field. Precisely the same result is obtained if a Harrison transformation with the appropriate value of the dilaton coupling is performed directly on the reduced solution. As a matter of fact, the equivalence between twisted KK reductions and Harrison transformations in the reduced KK theory was proven in [22]. In the case at hand, we know that the effect of performing a Harrison transformation on the metric (4.9) is just to replace T_1 by Λ_1 in (4.9).

4.2 The strings and membranes stretched between branes and anti-branes

The proper length of a string stretched between the poles where branes intersect depends in an essential way on the number of branes that intersect. These strings stretch along the line $r = \gamma$, parametrized by θ , $0 < \theta < \pi$. If there are less than four branes at the intersection then the proper spatial distance between poles is finite, but if all four branes are present then this distance is infinite. A situation where things can be studied further is that where a IIA configuration can be uplifted to $D = 11$ supergravity. The line $r = \gamma$ is fibered with the extra dimensions and becomes a surface. As a consequence, the string stretching between branes becomes a membrane. For the case of the $D6-\overline{D6}$ the study of such a membrane was carried out in [6]. The configuration in (3) in the previous subsection is also suitable for such an analysis, and will allow us to recover as a particular case the results of [6].

When we uplift (4.7) to eleven dimensions we obtain a Kaluza-Klein dipole superposed to a system of three intersecting delocalized $M2$ branes and three intersecting delocalized $M2$ anti-branes, that is,

$$\begin{aligned}
ds_{11}^2 = & (T_2 T_3 T_4)^{2/3} \left[-dt^2 + \frac{\Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4}{(r^2 - \gamma^2 \cos^2 \theta)^3} \left(\frac{dr^2}{\Delta} + d\theta^2 \right) \right] \\
& + \frac{\Delta \sin^2 \theta}{T_1 (T_2 T_3 T_4)^{1/3}} d\varphi^2 + \frac{T_1}{(T_2 T_3 T_4)^{1/3}} (dx_{11} - 2A_\varphi d\varphi)^2 \\
& + \frac{T_2^{2/3}}{T_3^{1/3} T_4^{1/3}} (dx_1^2 + dx_2^2) + \frac{T_3^{2/3}}{T_2^{1/3} T_4^{1/3}} (dx_3^2 + dx_4^2) + \frac{T_4^{2/3}}{T_2^{1/3} T_3^{1/3}} (dx_5^2 + dx_6^2). \quad (4.13)
\end{aligned}$$

In M-theory, the open string state should be described by a membrane wrapped on the surface $r = \gamma$. In order to distinguish it from the other, self-gravitating $M2$ -branes in the configuration, we will denote this one as an $m2$ -brane. As we just said, it wraps the surface $r = \gamma$. This surface is a bolt of the Killing vector $\mathbf{q} = \partial/\partial x_{11} + \frac{a_1}{2M_1(M_1+\gamma)} \partial/\partial \varphi$. It is convenient to work with an “adapted” coordinate $\phi = \varphi - \frac{a_1}{2M_1(M_1+\gamma)} x_{11}$ such that $\mathbf{q}\phi = 0$. In terms of this coordinate the metric induced on the bolt is given by

$$\begin{aligned}
ds_B^2 = & \prod_{i=2}^4 \left[a_i^4 \left(\frac{2q_i}{\gamma} + \frac{a_i^2 \sin^2 \theta}{\gamma^2} \right) \right]^{1/3} \frac{(\gamma + M_1)^2 - a_1^2 \cos^2 \theta}{\gamma^4 \sin^2 \theta} d\theta^2 \\
& + \prod_{i=2}^4 \left[\frac{\gamma^2}{a_i^2} \left(\frac{2q_i}{\gamma} + \frac{a_i^2 \sin^2 \theta}{\gamma^2} \right) \right]^{1/3} \frac{4M_1^2(\gamma + M_1)^2}{(\gamma + M_1)^2 - a_1^2 \cos^2 \theta} d\phi^2, \quad (4.14)
\end{aligned}$$

Now, when all four charges are turned on, this surface is topologically a cylinder. Its shape, and therefore that of the $m2$ -brane that wraps it, is like a sphere with two infinite funnels at its poles. This is most easily understood by looking at the geometry near the poles in $D = 11$,

after changing to the coordinates (2.9). We know that near the poles the geometry is, up to some angular distortion, the same as that of the core of an intersection between a KK monopole and three M2-branes. But the latter is

$$ds_{11}^2 = -dt^2 + \sum_{i=1}^6 dx_i^2 + \frac{(q_2 q_3 q_4)^{1/3}}{q_1} (dx_{11} + q_1 \cos \bar{\theta} d\varphi)^2 + (q_2 q_3 q_4)^{1/3} q_1 \left(\frac{d\rho^2}{\rho^2} + d\bar{\theta}^2 + \sin^2 \bar{\theta} d\varphi^2 \right). \quad (4.15)$$

and we explicitly see that $\rho = 0$ is down an infinite funnel of constant curvature fibered with x_{11} (there will be some angular distortion in the situation at hand, though).

The proper area of the bolt

$$A = \int d\theta d\phi \sqrt{g_{\theta\theta} g_{\phi\phi}} = 2M_1(\gamma + M_1) \int d\theta d\phi \prod_{i=2}^4 \left(\frac{2a_i q_i}{\gamma^2} + \frac{a_i^3 \sin^2 \theta}{\gamma^3} \right)^{1/3} \frac{1}{\sin \theta}, \quad (4.16)$$

is infinite due to the divergence of the integration at $\theta = 0, \pi$. Since the energy of the $m2$ -brane is $E = T_{m2} A$, where T_{m2} is the membrane tension, we reach the conclusion that the energy of the $m2$ -brane stretched inbetween the poles is infinite! Notice that it remains infinite even if we set $a_1 = 0$ (so that $\gamma = M_1$). If the latter is to be considered as the limit of coincidence of the branes and antibranes, then the conclusion is even more striking than that reached in [6].

The situation, however, is different if one sets one, two or three $M2$ charges to zero in (4.14). Say that n of these charges are different from zero. Then, neglecting the angular distortion, the geometry near the poles is

$$ds_{11}^2 = -dt^2 + \sum_{i=1}^6 dx_i^2 + \frac{\tilde{q}^{1/3}}{q_1} \rho^{1-n/3} (dx_{11} + q_1 \cos \bar{\theta} d\varphi)^2 + \tilde{q}^{1/3} q_1 \rho^{-1-n/3} (d\rho^2 + \rho^2 (d\bar{\theta}^2 + \sin^2 \bar{\theta} d\varphi^2)). \quad (4.17)$$

where \tilde{q} is the product of the non-zero M2 charges. It is straightforward to see that if $n \neq 0, 3$, the geometry is singular. When $n = 0$ the geometry is just $\mathbf{R}^{10,1}$, since this is the core of a KK monopole. The singularity for $n = 1, 2$ is nevertheless a finite spatial distance away, so the $m2$ -brane is topologically a sphere, with curvature singularities at the poles. Its proper area A , and the energy of the $m2$ -brane, is finite.

Let us now consider the limit where the branes and antibranes are coincident. It is not obvious what choice of parameters in the solution should correspond to this limit. One would certainly require any external fields to be absent, since their effect is to pull apart the poles. For the $D6-\overline{D6}$ system it was argued in [6] that one should also require $a = 0$, since this minimizes the distance between poles. In the present case, if we set all the a_i to zero then

we would have all M_i equal to each other, and as a consequence all charges would be equal, which seems too restrictive. In order to motivate other alternatives, notice first that all the branes at an intersection move together, since they all intersect at the same pole. Then, it might be enough to set just one of the a_i to zero. This would leave us with four parameters, which can be regarded as the four charges of the branes, as desired. Since a_1 is singled out as characterizing the twist in the eleventh direction, then, at least in the context of M-theory, it probably makes more sense to define the coincidence limit by setting only $a_1 = 0$ (so that $\gamma = M_1$), while leaving a_2, a_3, a_4 arbitrary.

With this choice, the conclusions we have reached above for the m2-brane stretched between branes and antibranes still hold in the coincidence limit: when any of the charges of the M2-branes is zero the proper area of the m2-brane is finite, whereas it is infinite when all charges are turned on. This conclusion, which may be taken as a ‘prediction’ about the strong coupling limit of this brane-antibrane configuration, is even more striking than that reached in [6] for $D6-\overline{D6}$ branes.

5 Discussion and Outlook

Although BPS states in string/M-theory have been under very close scrutiny over the last few years, they are far from providing us with the full spectrum of such theories. For instance, Sen has recently highlighted the rôle played by D-brane—anti-D-brane systems in constructing both unstable and stable non-BPS D-branes [23]. Motivated by his earlier work [6] on a $D6-\overline{D6}$ configuration of type IIA string theory, which he relates to the Kaluza-Klein dipole solution [4] (Euclidean 4d Kerr metric) embedded in eleven dimensions, and also by the recent work of one of us [7] on black diholes, we have identified new classical exact solutions to 4d General Relativity containing n abelian gauge fields and $n - 1$ independent scalar fields, whose generic lagrangians are given by (3.24). As already noted in the introduction, the case of two electric and two magnetic charges ($n = 4$) is particularly interesting since the corresponding lagrangian (1.1) arises as a consistent truncation of a wide class of low energy superstring compactifications, and therefore, the new 4d solutions can be uplifted and interpreted as rather sophisticated brane—anti-brane systems.

For general values of n , our 4d solutions depend on $n + 1$ parameters $\{\gamma, a_i, i = 1, \dots, n\}$, are static, axisymmetric and describe composite diholes. Indeed, the near horizon analysis of these solutions reveals they contain two throats which one can identify, for arbitrary values of the parameters (which label the n charges and the separation between the holes), with the throats of two oppositely charged extremal composite black holes. Although the composite dihole configurations are plagued with conical singularities, it is possible to suspend them in external

magnetic fields via generalised Harrison transformations, and tune these fields to values which eliminate the conical deficit and keep the configurations in equilibrium, though unstable: a slight deviation along the symmetry axis from the equilibrium configuration is expected [5] to result in either making the composite black holes collapse onto each other or accelerate apart. This instability, however, is of a completely different nature from the tachyonic instability appearing in perturbative string theory [23]. One might even wonder if the latter is a feature that survives when the string coupling is increased and the effects of self-gravity become important.

The total charge of the $U(1)^4$ composite diholes is zero while their ADM mass, whether or not they are suspended in external fields, is given by $E = \frac{1}{2} \sum_{i=1}^4 M_i = \frac{1}{2} \sum_i (\gamma^2 - a_i^2)^{1/2}$ and is generically strictly positive.⁶ These configurations are therefore non extremal, and when analysed in a context of supersymmetry, break all supersymmetries, an observation which is particularly obvious in the presence of external fields, which are asymptotically Melvin and therefore have no Killing spinors associated to them. Another interesting observation is that the mass of the two composite extremal black holes, which is equal to $2M_{bh} = \frac{1}{2} \sum_{i=1}^4 \frac{M_i(M_i + \gamma)}{\gamma}$, exceeds the ADM mass E of the composite dihole: the latter is therefore non-marginally bound. When uplifted to ten or eleven dimensions, it becomes a supergravity soliton and can be interpreted as a system of four intersecting branes and four intersecting anti-branes as sketched in Figure 1, with the (anti)branes localized in the overall transverse directions only. Each brane is charged under a different $U(1)$ and has its corresponding anti-brane parallel to it. The existence of such configurations of branes and antibranes at arbitrary (large) separation 2γ when $a_i \gg M_i$ is another indication that the static force vanishes between the branes and antibranes.

We thus succeeded in providing classical solutions to supergravity theories which describe static, zero charge configurations which appear as a cluster of intersecting charged branes and a cluster of intersecting charged anti-branes. They are non-trivial generalisations of the $D6-\overline{D6}$ systems analysed by Sen, who recognised that systems with *coincident* branes and anti-branes could in particular be used in the construction of stable non-BPS states of a new type, via orientifolding and orbifolding. His analysis relies on the perturbative description of D-branes, and it would be very instructive to study whether one can deform the supergravity solitons associated to brane—anti-brane systems in such a way that the resulting solutions (if any) describe these new stable non-BPS states. A first step in this direction should involve the study of the coincidence limit of the branes and antibranes in configurations of the sort we have been discussing. However, it appears that in the presence of gravity these systems exhibit features markedly different to those seen at the perturbative level, in particular, the size of membranes (and strings) stretched between the branes and antibranes remains non-zero (even infinite) in the limit of coincidence. This may cast some doubt on the possibility of carrying

⁶We are not considering the possibility of diholes made out of ‘massless’ black holes.

out comparisons between calculations performed at the weakly (CFT) and strongly coupled (supergravity) regimes.

Acknowledgements

We would like to thank Ruth Gregory and Simon Ross for conversations and comments on the manuscript. AC is supported by the Royal Thai Government through DPST scholarship. RE is supported by EPSRC through grant GR/L38158 (UK), and by grant UPV 063.310-EB187/98 (Spain). AT acknowledges the Leverhulme Trust for a Fellowship.

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